

Quark and Lepton Flavor Physics from F-Theory

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April 9, 2009

Abstract

Recent work on local F-theory models shows the potential for new categories of flavor models. In this paper we investigate the perturbative effective theory interpretation of this result. We also show how to extend the model to the neutrino sector.

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1 Introduction

The well-known flavor problem has perplexed particle theorists for quite some time now. With each new measurement the mysteries seem to increase. For example, measurements of neutrino masses to date have defied expectations, with large mixing angles and at least two of the neutrinos significantly more degenerate than in the quark sector. Turned around, the patterns of masses and mixings in the quark and lepton sector could be providing some important clues about underlying physics – physics that admittedly is often hard to test.

Of course there are many possible flavor models at this point, and it is difficult to distinguish among them. Only a few are elegant enough to avoid requiring many new arbitrary charges or parameters. In this sense higher-dimensional models seem promising, in that wavefunctions can naturally account for hierarchies and angles [1, 2, 3, 4]. Also many predictions in the end rely only on the nature of the wavefunctions in higher-dimensional space. The particular types of wavefunctions and Yukawas for the models discussed in [5] and in this paper use the existence of models based on 7-branes in a significant way, as we will discuss and expand on.

In this paper we consider the recent F-theory models of flavor proposed in [5, 6, 7]. We try to identify the distinguishing features that might make these models special. We show the models do have a low-energy effective field theory interpretation in a Froggatt-Nielsen-like form but that to truly reproduce the F-theory predictions would require a model that looks inelegant from a low-energy point of view. The presence of KK modes in the higher-dimensional theory automatically provides extra flavor-carrying states whose presence influences the structure of the low-energy mass matrices.

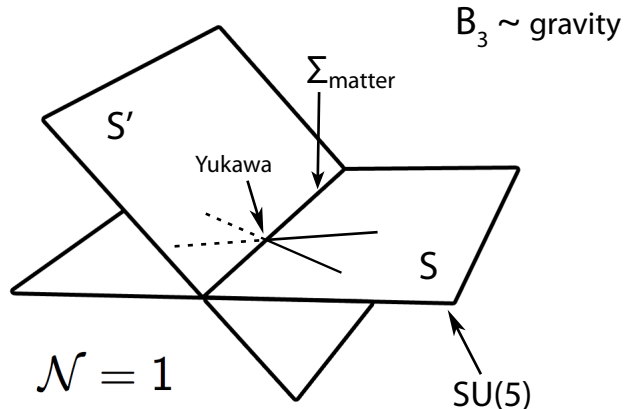
We also point out the qualitative features we find most important about these models. One is that they predict approximately rank-one matrices for both up and down quark Yukawas, a prediction that seems supported by what we know. Furthermore we argue that the most natural implementation of neutrinos would involve an approximately rank-two matrix, predicting a third small eigenvalue. Finally we argue that the framework is sufficiently general to support the measured mixing angles, though we find predictions of these parameters less robust, but completely compatible with known numbers.

2 Review of F-theory GUTs

We now review the set-up of Refs. [5, 6, 7]. At and below the GUT scale, an F-theory GUT is a higher dimensional QFT whose degrees of freedom live on submanifolds of a complex 3-fold B_3 (times $\mathbb{R}^{1,3}$). Gravity lives in the bulk, and we'll assume the decoupling limit of [6], where $M_{pl} \rightarrow \infty$. 7-branes with world-volume $\mathcal{N} = 1$ SUSY gauge theories are wrapped on complex surfaces¹ $S, S', \dots \subset B_3$. The associated gauge groups $G_S, G_{S'}, \dots$ depend

¹2-complex dimensional, or 4-real dimensional.

on the number and type of 7-branes wrapping each surface.² Matter resides in 1-complex dimensional intersections of two such surfaces, $\Sigma = S \cap S'$, called “matter curves.”



dim.	internal dim.	feature
10	$6 = \dim(B_3)$	gravity
8	$4 = \dim(S)$	gauge fields
6	$2 = \dim(S \cap S')$	matter
4	$0 = \dim(S \cap S' \cap S'')$	interactions

Figure 1: The structure of an F-theory GUT

We can identify the matter localized on an intersection of two branes $\Sigma \subset S \cap S'$ by beginning with both branes on top of each other (which has a simple description as a “parent” 8-dimensional effective gauge theory), and then rotating S' off of S by turning on a linear vev for the field φ representing the transverse distance between the branes. Degrees of freedom trapped near $\varphi = 0$ make up the theory in the matter curve $S \cap S'$. This Riemann surface supports vector-like matter charged under $G_S \times G_{S'}$.

A critical distinction for these models is that several distinct flavors can live on a single such matter curve. An index theorem determines the number of massless modes that reside on a Riemann surface formed by the intersection of S and S' . This allows for the possibility of three zero-mode solutions on these surfaces, each with its own independent wavefunction. This is not generally the case for models based on brane intersections, where generally multiple generations exist on the same surface only when multiple branes coincide, in which case the three generations would have the same gauge charges and wavefunctions, making hierarchies difficult to establish geometrically. In the F-theory set-up, the three generations can participate differently in the Yukawas, and this is in fact generally the case. This is the

²More precisely, in F-theory, the 3-fold B_3 , the surfaces S, S' , and the types of branes wrapping those surfaces are all encoded in the geometric data of a compact elliptically fibered Calabi-Yau 4-fold. For instance, 7-branes wrap loci where the elliptic fiber of the 4-fold degenerates, and the associated gauge group is specified by the singularity type of the fibration along the brane. In this paper, we won’t need most of these details, and it will suffice to take the 7-branes and their associated field theories (derived in [6]) as an effective description.

critical feature that allows for an interesting geometrical generation of the mass hierarchies and mixing angles.

The Yukawa couplings responsible for masses come from superpotential interactions between matter fields arising on point-like intersections of the associated matter curves. Because these matter representations are the residue of “parent” gauge interactions that are broken when the 7-branes do not coincide, the Yukawa interactions can be thought of as arising from gauge interactions and supersymmetry. Because the intersecting branes are in different orientations, the associated gauge bosons are heavy, so the Yukawas survive without additional spurious gauge interactions.

In the situations considered in [5, 6, 7], the geometry of the GUT brane S precludes the existence of bulk zero modes resembling SM matter. Therefore SM-like quark Yukawas require three additional surfaces to intersect S at a single point. We note here that although three surfaces intersecting is generic, four surfaces is not and we view this as an additional assumption. However, if you view an exceptional gauge group in the parent theory as fundamental, and the surfaces as arising from breaking the gauge symmetry, the Yukawa and four-way intersection is automatic [7]. This is entirely possible in the local description but it is ultimately important to see if this assumption will be realized in a full global model [8].

At this point we already see that Yukawa matrices are approximately rank-one, since the intersection of the three matter curves on S occurs only at a single point in the internal dimensions, and the Yukawa is roughly the outer product of the three left- and right-handed wavefunction vectors. This goes a long way toward realizing the structure we see in the quark masses. We will soon see that with the true wavefunctions, one finds the hierarchical mass matrices we know to describe the quark sector.

2.1 Matter Zero Mode Wavefunctions

It will be important for us to understand in more detail how SM matter arises when branes are rotated apart starting from a parent gauge theory, and what determines the zero mode wavefunctions. We’ll quickly review the discussion in [6], itself a review of [9].

Suppose that S has some GUT group G_S (which for concreteness we can take to be $SU(5)$), S' has gauge group $U(1)$, and the parent theory has gauge group G , broken to $G_S \times U(1)$ by turning on a generator $T \in \text{Ad}_G$. The degrees of freedom in the parent gauge theory (with $\mathcal{N} = 1$ superpartners paired up) are

$$A_\mu, \eta_\alpha \quad \text{scalars on } S, \text{ in } \text{Ad}_G \quad (1)$$

$$A_{\bar{m}}, \psi_{\alpha\bar{m}} \quad (0,1)\text{-forms on } S, \text{ in } \text{Ad}_G \quad (2)$$

$$\varphi_{mn}, \chi_{\alpha mn} \quad (2,0)\text{-forms on } S, \text{ in } \text{Ad}_G \quad (3)$$

and their complex conjugates. For the moment, take S to be \mathbb{C}^2 with coordinates z_1, z_2 . The field φ represents transverse directions to the brane, so rotating away S' corresponds to

giving φ a linear vev proportional to T :

$$\varphi = m^2 z_1 T dz_1 \wedge dz_2 \quad (4)$$

The mass scale m^2 is related to a characteristic (stringy) scale M_* of the F-theory compactification, and the angle θ between the branes S, S' . The line $z_1 = 0$ is the matter curve $\Sigma = S \cap S'$. By supersymmetry, to find the matter on Σ , it suffices to look just for fermionic degrees of freedom localized near $z_1 = 0$.

The action for fermions in the parent theory is

$$I_S = \int_{\mathbb{R}^{3,1} \times S} d^4x \operatorname{Tr} \left(\chi^\alpha \wedge \bar{\partial}_A \psi_\alpha + 2i\sqrt{2}\omega \wedge \partial_A \eta^\alpha \wedge \psi_\alpha + \frac{1}{2} \psi^\alpha [\varphi, \psi_\alpha] + \sqrt{2} \eta^\alpha [\bar{\varphi}, \chi_\alpha] \right) + \text{h.c.} + \text{kinetic terms} \quad (5)$$

where $\omega = \frac{i}{2} g_{i\bar{j}} dz^i \wedge d\bar{z}^{\bar{j}}$ is the Kahler form on S , and $\partial_A = dz^m \left(\frac{\partial}{\partial z^m} + A_m \right)$. Varying with respect to η and ψ gives the zero mode equations

$$\omega \wedge \partial_A \psi^\alpha + \frac{i}{2} [\bar{\varphi}, \chi^\alpha] = 0, \quad (6)$$

$$\bar{\partial}_A \chi^\alpha - [\varphi, \psi^\alpha] = 0 \quad (7)$$

The modes that get trapped near $z_1 = 0$ are linear combinations of ψ, χ , and η and have a nonzero charge under the adjoint action of T . The solutions are easy to find in the absence of flux ($A = 0$, in an appropriate gauge). They are

$$\psi_{\bar{2}} = 0, \quad \psi_{\bar{1}}, \chi_{12} \propto \alpha(z_2) e^{-|mz_1|^2} \quad (8)$$

where $\alpha(z_2)$ is holomorphic.

The presence of flux deforms the wave functions to

$$\psi_{\bar{1}}, \chi_{12} \propto \alpha(z_2) e^{-|mz_1|^2} \exp \mathcal{M}(z, \bar{z}) \quad (9)$$

where $\mathcal{M}(z, \bar{z})$ is a (not necessarily holomorphic) function that vanishes when the flux vanishes. For instance in a constant background flux, $A = -F_{1\bar{1}} \bar{z}_1 dz_1 + F_{2\bar{2}} z_2 d\bar{z}_2$,³

$$\mathcal{M}(z, \bar{z}) = -F_{2\bar{2}} z_2 \bar{z}_2 + \frac{1}{2} F_{1\bar{1}} z_1 \bar{z}_1 + \dots \quad (10)$$

³Note for instance that positive flux $F_{2\bar{2}}$ through the curve $z_1 = 0$ causes wavefunctions to decay rapidly away from $z_2 = 0$. So, wavefunctions on Σ are “attracted” to regions of positive flux, which is related to the well-known fact that the number of normalizable zero modes on Σ is equal to the total flux through Σ .

3 Review of Yukawa Calculation

The superpotential of the parent theory is

$$W = M_*^4 \int_S \text{Tr}(\mathbf{F}^{(0,2)} \wedge \mathbf{\Phi}) = M_*^4 \int_S \text{Tr}(\mathbf{A} \wedge \mathbf{A} \wedge \mathbf{\Phi}) + \text{quadratic} \quad (11)$$

where $\mathbf{\Phi} = \varphi + \theta\chi + \dots$, $\mathbf{A} = A + \theta\psi + \dots$. Since the zero modes are linear combinations of variations of the superfields \mathbf{A} and $\mathbf{\Phi}$, the cubic term in (11) gives rise to Yukawa couplings proportional to structure constants in the parent gauge group and the wavefunction overlap of matter fields.

For example, consider the down Yukawa in an $SU(5)$ GUT coming from an intersection of the matter curves $\Sigma_Q, \Sigma_D, \Sigma_{H_d} \subset S$ at a point p_d . We can choose coordinates z_1, z_2 near p_d such that Σ_Q and Σ_D , are the zero loci of z_2, z_1 , respectively. For concreteness, we'll take the curve Σ_{H_d} to be $z_1 = z_2$ (Fig. 2).

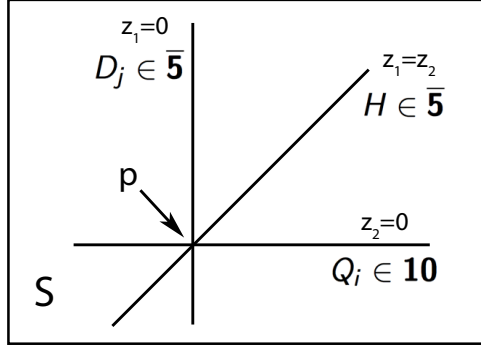


Figure 2: An intersection of matter curves giving the down Yukawa.

Then the zero modes have wavefunctions

$$Q_i \sim \alpha_i(z_1) e^{-|m_1 z_2|^2 + \mathcal{M}_1(z, \bar{z})} \quad (12)$$

$$D_j \sim \beta_j(z_2) e^{-|m_2 z_1|^2 + \mathcal{M}_2(z, \bar{z})} \quad (13)$$

$$H_d \sim \gamma(z_1 + z_2) e^{-|m_3(z_1 - z_2)|^2 + \mathcal{M}_3(z, \bar{z})} \quad (14)$$

where the α_i, β_j , and γ are holomorphic. Following [5], by performing unitary flavor rotations we can require

$$\alpha_i(z_1) = \left(\frac{z_1}{R_1} \right)^{3-i} + \text{lower order} \quad (15)$$

$$\beta_j(z_2) = \left(\frac{z_2}{R_2} \right)^{3-j} + \text{lower order} \quad (16)$$

$$\gamma(z) = \text{const.} + \text{lower order} \quad (17)$$

where R_1, R_2 are roughly the sizes of the matter curves Σ_Q, Σ_D . Our Yukawa is proportional to the wavefunction overlap

$$Y_{ij} \propto M_*^4 \int d^2 z_1 d^2 z_2 \left(\frac{z_1}{R_1} \right)^{3-i} \left(\frac{z_2}{R_2} \right)^{3-j} \exp(-|mz|^2 + \mathcal{M}(z, \bar{z})) \quad (18)$$

where $|mz|^2$ is short for $|m_1 z_2|^2 + |m_2 z_1|^2 + |m_3(z_1 - z_2)|^2$, and $\mathcal{M}(z, \bar{z})$ is the sum of the flux-dependent distortions \mathcal{M}_i for each curve.

Since the Gaussian measure $d^2 z_1 d^2 z_2 e^{-|mz|^2}$ is invariant under $U(1)$ rotations of each coordinate, to get a nonzero result we need to pull down sufficient powers of \bar{z} in the Taylor expansion

$$\mathcal{M}(z, \bar{z}) = \sum \mathcal{M}^{a,b} \left(\frac{\bar{z}_1}{R_1} \right)^a \left(\frac{\bar{z}_2}{R_2} \right)^b + \text{holomorphic and mixed terms} \quad (19)$$

We'll approximate $\mathcal{M}^{a,b} \sim \mathcal{M}_0$, where \mathcal{M}_0 is the characteristic size of the distortion \mathcal{M} . This is equivalent to assuming that the distortion factor varies by order \mathcal{M}_0 over curves of sizes R_1 and R_2 . The authors of [5] identify two types of expansions of the exponential $e^{\mathcal{M}}$, which are important in different limits. The “derivative” expansion brings down the single term $\mathcal{M}^{3-i,3-j} \bar{z}_1^{3-i} \bar{z}_2^{3-j}$ in (19) necessary to make the integrand $U(1)$ -invariant, giving

$$Y_{ij}^{\text{DER}} \propto \frac{M_*^4}{m^4} \left(\frac{1}{m^2 R_1^2} \right)^{3-i} \left(\frac{1}{m^2 R_2^2} \right)^{3-j} \mathcal{M}_0 \quad (20)$$

The “flux” expansion brings down multiple powers of $\mathcal{M}^{1,0} \bar{z}_1$ and $\mathcal{M}^{0,1} \bar{z}_2$, giving

$$Y_{ij}^{\text{FLX}} \propto \frac{M_*^4}{m^4} \left(\frac{\mathcal{M}_0}{m^2 R_1^2} \right)^{3-i} \left(\frac{\mathcal{M}_0}{m^2 R_2^2} \right)^{3-j} \quad (21)$$

Assuming that $m \sim M_*$ and $R_1 \sim R_2 \sim R$, we see that the small parameters relevant for the hierarchy in the Yukawas are

$$\kappa = \frac{1}{m^2 R^2}, \quad \epsilon^2 = \mathcal{M}_0 \kappa \quad (22)$$

and we have

$$Y = Y^{\text{FLX}} + Y^{\text{DER}} = \begin{pmatrix} \epsilon^8 & \epsilon^6 & \epsilon^4 \\ \epsilon^6 & \epsilon^4 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix} + \frac{\epsilon^2}{\kappa} \begin{pmatrix} \kappa^4 & \kappa^3 & \kappa^2 \\ \kappa^3 & \kappa^2 & \kappa \\ \kappa^2 & \kappa & 1 \end{pmatrix} \quad (23)$$

The relation $(RM_*)^4 = \alpha_{\text{GUT}}^{-1}$ implies

$$\epsilon \sim \kappa \sim \alpha_{\text{GUT}}^{1/2}, \quad (24)$$

from which the authors of [5] show that the matrices in (23) can reproduce the known quark masses.⁴ Further, assuming that both the up and down Yukawas take the form (23) with

⁴The success of this estimate relies on the assumption $m \sim M_*$. This is equivalent to the assumption that oscillations of matter fields transverse to a matter curve decouple at a stringy scale, which is necessary for us to think about matter as being localized on curves in the first place. However, the order one ratio m/M_* could affect the hierarchy in 23.

Since there are order one parameters in each entry, this is not really an issue. But the best fit from, for instance, the authors of [5] for up quark masses using the Y^{FLX} hierarchy $1 : \epsilon^4 : \epsilon^8$ requires $\epsilon \sim 0.26$, which is a bit larger than $\alpha_{\text{GUT}}^{1/2} \sim 0.2$. Fitting quark masses at the GUT scale requires an even bigger ϵ . Below, when we discuss neutrino masses in terms of α_{GUT} , we will only be able to estimate up to similar order one factors.

the same basis for the left-handed quarks, they show that the resulting quark mixing angles agree nicely with V^{CKM} in the Standard Model. We'll return to this assumption in Section 5.

4 Effective Field Theory Interpretation

Notice that the matrix Y^{FLX} takes the typical single-field Froggatt-Nielsen (FN) form [10] with $U(1)$ flavor charges 4, 2, 0 for generations 1, 2, 3, respectively, and a spurion field ϵ with flavor charge -1 . However, Y^{DER} differs from a usual FN structure by the small parameter $\frac{\epsilon^2}{\kappa}$ out front. If we wanted to reproduce only the DER matrix, we could account for it in an FN scenario by assigning the Higgs a charge. However, in that case the large entry Y_{33}^{FLX} would be forbidden. Clearly, without additional fields (other than the zero-mode Higgs and three light generations of left- and right-handed fields), one cannot generate the sum of the two types of matrices with symmetries alone. One can take simplified cases where we can simply generate both matrices (see below), but our goal here is to write down the minimal theory that directly describes how the geometry produces (23).

Perhaps the simplest way to find the field theory that reproduces the F-theory result is to explicitly do perturbation theory in the flux from the beginning. Separate the background gauge field A as $A = A_0 + a$, where A_0 is in the same topological class as A , but has zero flux near p_d . In the A_0 background, the zero modes have the simple holomorphic \times gaussian form (12-14) near p_d , without the distortion factors \mathcal{M} . The resulting Yukawa is that of (23) with $\epsilon = \kappa = 0$, namely a rank-1 matrix involving only the third generations.

The perturbation a restores the flux near p_d , and somehow leads to mixing between generations, and thus corrections to this rank-1 Yukawa. The action (5) involves only mass mixing so the obvious interpretation of kinetic mixing among generations is ruled out (though it would be permitted with high-dimension terms included in the Lagrangian). But mass mixing among the generations is of course not permitted in a chiral theory without insertion of a Higgs field. So the only possible interpretation is in terms of mass mixing between zero modes and KK modes with the same gauge quantum numbers. (Note that we use the term KK mode for all the heavy modes, including the orthogonal combinations of η , χ , and ψ .) So we can think of a as distorting zero mode wavefunctions, or equivalently mixing zero modes of the A_0 background with KK modes of the A_0 background.

However, this still leaves the question of where the corrections to the rank-1 Yukawa arise: if only mass mixing played a role, all the correction terms would involve at least two powers – rather than a single power – of the flux $\mathcal{M}_0 = \frac{\epsilon^2}{\kappa}$. We now show that the KK modes have nonzero Yukawa couplings to the different generations, and these Yukawas in combination with the mixing via a are the source of the perturbation to the rank-1 Yukawa.

4.1 Feynman Rules for KK Modes

To make this picture precise, let's first concentrate on the left-handed quark superfields $Q_i \in \mathbf{10}$. The Q_i are zero modes of a vector-like field $(Q, Q^c) \in \mathbf{10} \oplus \overline{\mathbf{10}} \subset \text{Ad}_G$ on $\mathbb{R}^{1,3} \times S$, which has KK expansion

$$Q(x, z, \bar{z}) = Q_i(x) f_i(z, \bar{z}) + \sum_I Q_I(x) g_I(z, \bar{z}) \quad (25)$$

$$Q^c(x, z, \bar{z}) = \sum_J Q_J^c(x) h_J(z, \bar{z}) \quad (26)$$

where the capital subscripts represent massive KK-modes. In the absence of flux near p_d , the zero mode wavefunctions f_i take the form $1, z_1, z_1^2$. For the sake of computing Yukawas, we'd like to classify the KK wavefunctions g_I in a similar way. Choose a basis $g_{a,b}^{c,d}$ such that

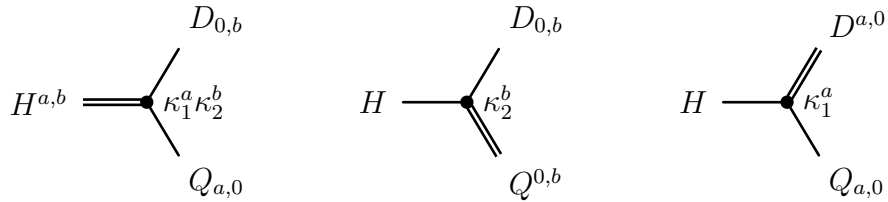
$$g_{a,b}^{c,d}(z, \bar{z}) \sim z_1^a z_2^b \bar{z}_1^c \bar{z}_2^d \quad \text{near } p_d. \quad (27)$$

We say that the fields $Q_{a,b}^{c,d}$ associated with $g_{a,b}^{c,d}$ have “ z -charge (a, b) ” and “ \bar{z} -charge (c, d) .” The zero modes are a special case: $Q_i = Q_{i,0}^{0,0}$. For the KK modes with z -charge $(0, 0)$, we'll use a simpler notation $Q^{c,d} = Q_{0,0}^{c,d}$.

Yukawa couplings are nonzero only when the powers of z_i are the same as the powers of \bar{z}_i . And each insertion of $z_i \bar{z}_i$ in the integral gives a factor $\kappa_i = \frac{1}{m^2 R_i^2}$. Thus, the allowed Yukawas are

$$H_{a_1, b_1}^{c_1, d_1} Q_{a_2, b_2}^{c_2, d_2} D_{a_3, b_3}^{c_3, d_3} \kappa_1^{a_1+a_2+a_3} \kappa_2^{b_1+b_2+b_3}, \quad \text{where } \sum a_i = \sum c_i \text{ and } \sum b_i = \sum d_i \quad (28)$$

For instance, at leading order in powers of z and \bar{z} , the Yukawas involving two zero modes and one KK mode are of the form



$$\quad (29)$$

The shapes of the wavefunctions enforce the relations between z and \bar{z} charges and the number of powers of κ_i . We can mimic this with flavor charges by declaring that κ_i have charges $-1, -1$ under z_i, \bar{z}_i , respectively. Then the couplings (28) are enforced by charge invariance. In other words, we can use $U(1)$ symmetries as a trick to get the right structure, but it really comes from the geometry.

Now let's combine this with the mixing from the perturbation a . We can read off the KK mass matrix from the terms bilinear in the fermions q_I, q_J^c associated with Q_I and Q_J^c . These are linear combinations of the fields ψ, χ, η in the parent theory, so M^{KK} comes from

fermion bilinear terms in the action (5) with gauge field A_0 :

$$I_{A_0} = \int_{\mathbb{R}^{3,1} \times S} d^4x \operatorname{Tr} \left(\chi^\alpha \wedge \bar{\partial}_{A_0} \psi_\alpha + 2i\sqrt{2}\omega \wedge \partial_{A_0} \eta^\alpha \wedge \psi_\alpha + \frac{1}{2} \psi^\alpha [\varphi, \psi_\alpha] + \sqrt{2} \eta^\alpha [\bar{\varphi}, \chi_\alpha] \right) + \text{h.c.} \quad (30)$$

$$= \int d^4x M_{JI}^{(\text{KK})} q_J^c q_I + \text{h.c.} \quad (31)$$

Similarly, the gauge field perturbation a couples to fermions as follows

$$I_a = \int_{\mathbb{R}^{3,1} \times S} d^4x \operatorname{Tr} \left(\chi^\alpha \wedge \bar{a} \wedge \psi_\alpha + 2i\sqrt{2}\omega \wedge a \wedge \eta^\alpha \psi_\alpha \right) + \text{h.c.} \quad (32)$$

$$= \int d^4x a_{JI} q_J^c q_I + a_{Ji} q_J^c q_i + \text{h.c.} \quad (33)$$

Note that while $M^{(\text{KK})}$ clearly only couples to KK modes, the perturbation a couples KK modes to each other, and also KK modes of \mathbb{Q}^c to zero modes of \mathbb{Q} . Including this mixing and integrating out the Q_J^c to lowest order in momentum induces a Yukawa coupling between zero modes alone

$$\overbrace{a_{Ji} Q_J^c Q_i} \quad Y_{Ij} H Q_I D_j = (M^{(\text{KK})})_{IJ}^{-1} a_{Ji} Y_{Ij} H Q_i D_j \quad (34)$$

The matrix

$$(M^{(\text{KK})})_{IJ}^{-1} a_{Ji} \quad (35)$$

is just multiplication by the distortion factor $\mathcal{M}(z, \bar{z})$ that comes from turning on a . This may be a bit surprising, since our notation has obscured the z dependence of all the wavefunctions, but it must be true since multiplication by $(M^{(\text{KK})})^{-1}a$ and $\mathcal{M}(z, \bar{z})$ both give the first order variation in the zero modes from turning on a . For instance, in our previous example (10) with matter curve $z_1 = 0$ in \mathbb{C}^2 , if we start with $A_0 = 0$, and turn on a constant flux $a = F_{2\bar{2}} z_2 d\bar{z}_2$, then

$$(M^{(\text{KK})})_{IJ}^{-1} a_{Ji} \sim \bar{\partial}_2^{-1} a = F_{2\bar{2}} z_2 \bar{z}_2 \quad (36)$$

which is indeed \mathcal{M} associated with constant flux $F_{2\bar{2}}$ through the curve. In the Taylor expansion (19), the derivatives $\mathcal{M}^{a,b}$ allow the zero modes to mix with KK modes that carry \bar{z} -charge (a, b) (in effect compensating for the apparent violation of \bar{z} charge that allows the Yukawas to be nonzero). Thus, we have the following diagrammatic rule for mixing to KK modes and propagating:

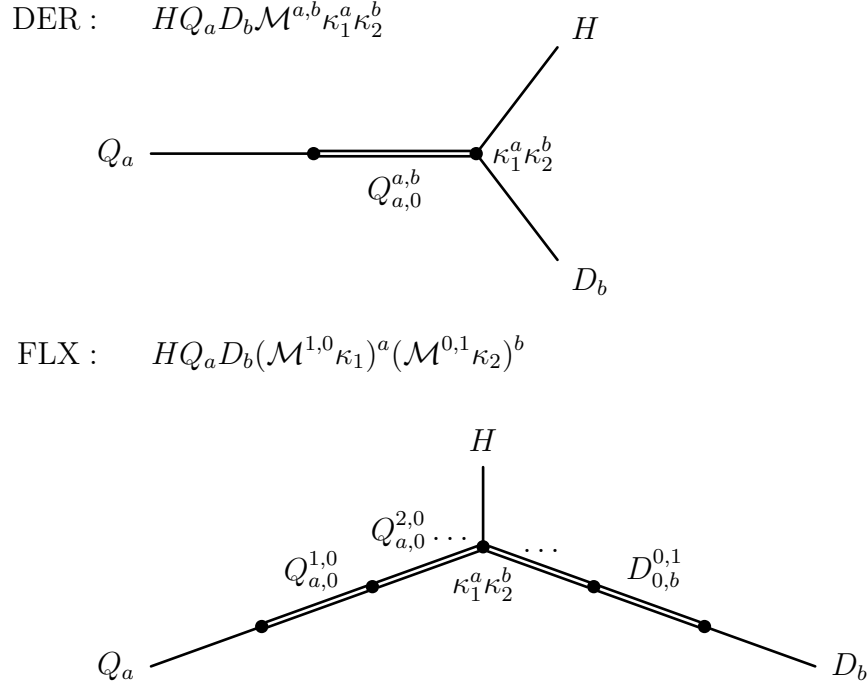
$$Q_a \text{ --- } \bullet \text{ --- } \overline{\overline{\hspace{1.5cm}}} \quad Q_{a,0}^{b,c} = \mathcal{M}^{b,c} \quad (37)$$

$$Q_{a,b}^{c,d} \text{ --- } \overline{\overline{\hspace{1.5cm}}} \bullet \text{ --- } \overline{\overline{\hspace{1.5cm}}} \quad Q_{a,b}^{c+e,d+f} = \mathcal{M}^{e,f} \quad (38)$$

where the factors $\mathcal{M}^{b,c}$ include both the vertex and the KK propagator, since those elements will appear together in all our Feynman diagrams.

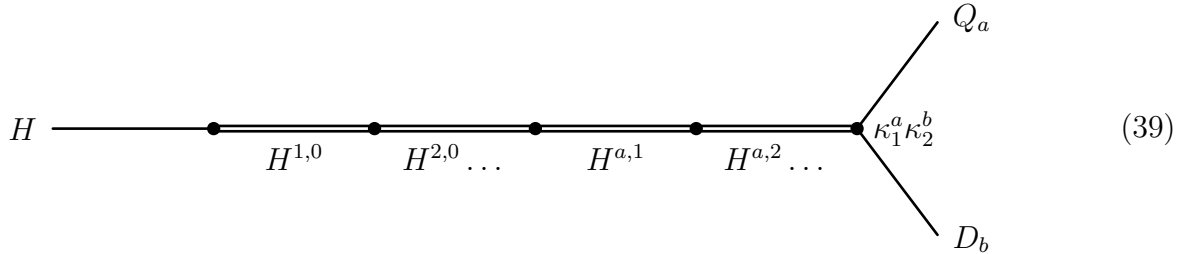
4.2 Diagrammatic Interpretation of Y^{DER} and Y^{FLX}

The two Yukawa textures Y^{DER} and Y^{FLX} arise naturally from the charge assignments, and have a diagrammatic interpretation in terms of mixing through KK modes



Note that in both cases, the zero modes carried the z charge that is also carried by the KK mode whereas the KK-modes (that mix with the zero modes) carry the \bar{z} charges that permit Yukawas to be nonvanishing.⁵

Of course, we also get corrections to the Yukawas through mixing with Higgs KK modes, in which case the KK modes carry purely \bar{z} charge. For instance, the flux interaction can come from



plus permutations of the internal KK mode lines.

Notice that if the expansion parameters ϵ^2 and $\kappa \sim \kappa_1 \sim \kappa_2$ were the same, we could have reproduced the sum $Y^{\text{FLX}} + Y^{\text{DER}}$ by introducing a single charged Higgs field, which

⁵The DER scenario is similar to shining in that a different spurion $\mathcal{M}^{a,b}$ is responsible for each entry of the matrix. However, the hierarchy in entries comes from z and \bar{z} charges, rather than signal propagation over extra dimensions.

could couple in the DER matrix while a neutral Higgs field coupled in the FLX matrix. However, ref. [5] matched quark masses with ϵ, κ as independent parameters. Furthermore, our goal here was to answer quite generally the question of effective field theory underlies the F-theory form found in [5].

We also note that in principle we could reproduce the above form with only a single generation of KK modes – either flavored ones for the Higgses or a single additional copy for all of the fermions. One can have a model with just such copies that reproduces the F-theory form, without the full KK structure and the full extra-dimensional theory.

4.3 Froggatt-Nielsen Type Models

We can account for the Yukawas purely in terms of charges and spurions (without additional Higgs or fermion fields) by integrating out the KK modes from our model. The result isn't pretty. We have four $U(1)$ charges (a, b, c, d) , counting the powers of $z_1, z_2, \bar{z}_1, \bar{z}_2$, respectively. We should think of $\mathcal{M}^{a,b}$ as spurions with charges $(0, 0, a, b)$, and κ_1 and κ_2 as spurions with charges $(-1, 0, -1, 0)$ and $(0, -1, 0, -1)$, respectively. The zero modes Q_a have charges $(a, 0, 0, 0)$ and the D_b have charges $(0, b, 0, 0)$.

In the situation where $\kappa_1 \approx \kappa_2$, we can simplify the charges and spurions while sacrificing some fidelity to the F-theory geometry. Specifically, we no longer need to distinguish between z_1 and z_2 , and we can just keep track of total z -charge and total \bar{z} -charge. We need only one spurion κ with charges $(-1, -1)$, and four spurions \mathcal{M}^a with charges $(0, a)$ for $a = 1, 2, 3, 4$. Both the right- and left-handed generations have charges $(i, 0)$ for $i = 0, 1, 2$. This is sufficient to reproduce both Yukawa textures of (23).

5 Mixing Matrices From Nearby Intersection Points

So far we have included only the perturbative contributions to the Yukawa couplings arising from flux-induced distortions of the wavefunctions. If the up and down Yukawas are generated at the same point, the resulting mixing matrix U_{same} agrees well with V^{CKM} in the Standard Model [5]. But in general the intersections occur at separated points p_u and p_d . Then, even though each individual Yukawa matrix is rank-one, the associated eigenvectors will in general be misaligned, yielding an additional contribution U_{sep} to the mixing matrix. Since we already know V^{CKM} agrees well with U_{same} , this yields a rough constraint on the separation $|p_u - p_d|$, which we now derive.

Around any point $p \in \Sigma_q$, we can pick coordinates z such that $z = 0$ at p and choose an orthonormal basis of zero modes f_p^i such that

$$f_p^i(z) \sim z^{3-i} + \text{lower order.} \quad (40)$$

The basis f_p^i is unique (up to $U(1)$ rotations of each element) at each point p . The matrix U_{sep} is the unitary rotation between the zero mode bases $f_{p_u}^i$ and $f_{p_d}^j$. If p_u and p_d are far

apart on the quark curve Σ_Q , these bases will in general be unrelated and U_{sep} will have order one angles. This is clearly undesirable in the quark sector where mixing angles are small.

The most obvious way to avoid this large mixing contribution is to require p_u be near p_d . In this case, we can say something nontrivial about U_{sep} by keeping track of the way the basis f_p varies with small changes in p . The key observation is that as we move from $z = 0$ to a nearby point, the functions 1 and z mix with each other at first order, as do the functions z and z^2 . However, 1 and z^2 do not mix at first order.

By requiring that the f_p^i vanish with the appropriate degree at p , we can derive the lower triangular part of an evolution equation for f_p^i :

$$\frac{\partial}{\partial p} \begin{pmatrix} f_p^3 \\ f_p^2 \\ f_p^1 \end{pmatrix} = \left(\begin{array}{ccc} 0 & ? & ? \\ -\frac{\partial_z f_p^2}{f_p^3} & 0 & ? \\ 0 & -\frac{\partial_z^2 f_p^1}{\partial_z f_p^2} & 0 \end{array} \right) \bigg|_{z=p} \begin{pmatrix} f_p^3 \\ f_p^2 \\ f_p^1 \end{pmatrix} \quad (41)$$

We've set the diagonal entries to zero because they would contribute just a $U(1)$ rotation on each basis element.⁶ Since the basis should remain orthogonal, we can fill in the upper triangular part with the requirement that the 3×3 matrix above be anti-Hermitian:⁷

$$\frac{\partial}{\partial p} \begin{pmatrix} f_p^3 \\ f_p^2 \\ f_p^1 \end{pmatrix} = \left(\begin{array}{ccc} 0 & \frac{\overline{\partial_z f_p^2}}{f_p^3} & 0 \\ -\frac{\partial_z f_p^2}{f_p^3} & 0 & \frac{\partial_z^2 f_p^1}{\partial_z f_p^2} \\ 0 & -\frac{\partial_z^2 f_p^1}{\partial_z f_p^2} & 0 \end{array} \right) \bigg|_{z=p} \begin{pmatrix} f_p^3 \\ f_p^2 \\ f_p^1 \end{pmatrix} \quad (42)$$

Finally, estimating $\partial_z \sim R^{-1}$, and integrating this over a short distance $\delta \cdot R$ gives the mixing matrix

$$U_{\text{sep}} \approx \begin{pmatrix} 1 & \delta & \delta^2 \\ \delta & 1 & \delta \\ \delta^2 & \delta & 1 \end{pmatrix} \quad (43)$$

Thus, nearby points have order δ mixings between adjacent generations, but order δ^2 mixings between the 1st and 3rd generations.⁸ This is not the structure of V^{CKM} in the standard model, and we'd like to ensure it gives a negligible contribution to mixing compared to U_{same} . The strongest bound on δ comes from the fact that the mixing between the second and third

⁶Note that $\partial_z f$ is gauge-covariant at a point where f vanishes, since $\partial_z f = (\partial_z + A_z)f$ at that point. Thus, the ratio $\frac{\partial_z f^2}{f^3}$ is well-defined. For the same reason, the second derivative $\partial_z^2 f^3$ is gauge-covariant, and the ratio $\frac{\partial_z^2 f^1}{\partial_z f^2}$ is well-defined.

⁷We can think of this equation as defining a $U(3)$ connection on the curve Σ_q . Since the basis f_p^i is unique up to $U(1)$ rotations of each basis element, the curvature of this connection should be contained within a $U(1)^3$ subgroup.

⁸This form (43) is familiar from the simplest Froggatt-Neilsen flavor models [10].

generations in the standard model is small $V_{23}^{CKM} \approx 0.04$. We should have $\delta \lesssim 0.04$, or $|p_u - p_d| \lesssim 0.04R$.⁹

Our constraint supports the observation in [5] that the quark sector shows either non-trivial fine tuning or perhaps evidence of a higher unification structure that forces p_u near p_d .

6 The Neutrino Sector

6.1 Mixing Angles for Dirac Neutrinos

Though dangerous with respect to quarks, the mixing matrix (43) is interesting from the perspective of Dirac neutrinos. Mixing angles for neutrinos follow a very different pattern than those for quarks. The current bounds are [11]

$$\sin^2(2\theta_{23}) > 0.92, \quad \sin^2(2\theta_{12}) = 0.86_{-0.04}^{+0.03}, \quad \sin^2(2\theta_{13}) < 0.19 \quad (44)$$

We first note that the angles in (43) take the basic form observed for neutrinos, with large mixing between adjacent generations and suppressed mixing between the first and third generations.¹⁰ Of course once p_u and p_d are sufficiently far away, δ is not small and all angles are of the same order. We can safely say that with distant points we predict large mixing angles (this point was also made in ref. [5]). We can also view (43) as suggesting that θ_{13} might be somewhat smaller if the Yukawas for leptons and neutrinos are generated at points that are not too far apart. These models would nonetheless be strongly disfavored if experiment determines that θ_{13} is substantially smaller than θ_{12}^2 . However, there are recent indications [12] of a lower bound on θ_{13} , and a best fit consistent with the relation $\theta_{13} \sim \theta_{12}^2$.

6.2 Neutrino Masses

We now turn to the question of the masses themselves. We know only neutrino mass-squared differences, but they already look very different from those in the the quark sector [11]

$$\Delta m_{21}^2 = (8.0 \pm 0.3) \times 10^{-5} \text{ eV}^2 \quad (45)$$

$$\Delta m_{32}^2 = (2.5 \pm 0.5) \times 10^{-3} \text{ eV}^2 \quad (46)$$

In addition to the obvious fact that neutrino masses are much smaller than either quark or charged lepton masses, the ratios of masses is also quite different.

Firstly, if neutrino masses fall into a normal hierarchy, with for instance $m_1 < m_2 < m_3$, then (46) suggests that the ratio m_3/m_2 is of order 5, not 25 like for quarks at the GUT scale. In this case, we would need to explain why the neutrino hierarchy is less steep. Secondly,

⁹This is a slightly stronger constraint than the rough estimate in [5]

¹⁰A single field Froggatt-Nielsen model could also give mixing angles with the texture (43).

it's possible that neutrino masses fall into an “inverted hierarchy,” with $m_1, m_2 \gg m_3$. This pattern would certainly not conform to the rank-one structure of the quark mass matrices.

Particularly in the case of an inverted hierarchy, though likely also in the case of the normal hierarchy given the mass ratios, we need a significantly different Yukawa structure from the F-theory motivated structures that we have encountered up to this point. We now argue that for an $SU(5)$ GUT, the singlet nature of the neutrino makes this a reasonable possibility. In particular, we show that whereas the quark matrix is expected to be rank-1, because the right-handed neutrino is a singlet, the neutrino mass matrix can generically have two large eigenvalues.

6.2.1 A Single Intersection Point

The fact that right-handed neutrinos N are gauge singlets means that the matter curve Σ_N must lie off the GUT brane S . By contrast, the Higgs H and leptons L have nontrivial gauge charges, so they live on curves inside S . Suppose L and H are localized on $\Sigma_L = S' \cap S$ and $\Sigma_H = S'' \cap S$, respectively. There is now the possibility that the singlet neutrinos live on $\Sigma_N = S' \cap S''$. In this case, each intersection point in $S \cap S' \cap S''$ would generate the Yukawa NHL . We'll assume that this is the case and that Σ_N supports three zero modes (Fig. 3).

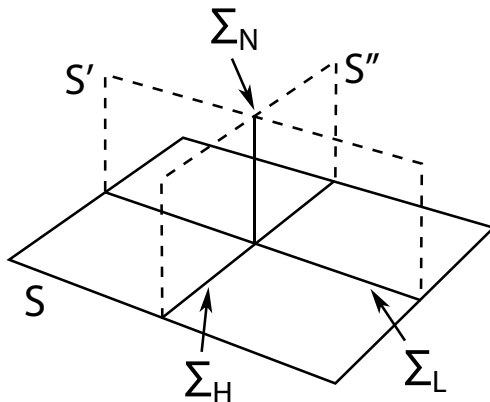


Figure 3: An approximately rank-1 Yukawa involving a gauge singlet.

If $S \cap S' \cap S''$ is a single point, we get an approximately rank-1 Dirac mass matrix m of the form Y^{FLX} or Y^{DER} , just like in the quark sector. This is difficult to reconcile with experiment, whether neutrinos are Majorana or Dirac. We briefly explain why, and then discuss a more viable setup.

If neutrinos are Dirac, we could explain their small masses via localization of right-handed neutrino wavefunctions away from the GUT brane S .¹¹ But then the neutrino mass matrix would generically have too large a hierarchy.

¹¹Based on the curvature of the geometry near S , it's possible to estimate the resulting exponential suppression in the size of the HNL coupling [7].

Suppose instead that N gets a large Majorana mass M , either from a self-intersection $\Sigma_N \cap \Sigma_N \cap \Sigma_\Theta$ with a scalar Θ that gets a vev, or (perhaps more interestingly) via $D3$ -instantons wrapping one of the branes that Σ_N lies in [13, 14].¹²

In the case of a Σ_N self-intersection at a point p , the matrix M will have eigenvalues of order $1, 1, \epsilon^2$. To see why, we first resolve the self-intersection at p , thinking of Σ_N as the image of a smooth curve $\tilde{\Sigma}_N$ with two points $p_1, p_2 \in \tilde{\Sigma}_N$ that both map to p . As before, we can choose a basis of wavefunctions $f_{p_1}^i$ that look like $1, z, z^2$ near p_1 . However, that uses up our freedom to redefine our basis, and generically all the $f_{p_1}^i$ will be constant to leading order at p_2 . Thus, in this basis the wavefunction overlap integral gives a symmetric matrix of the form

$$M \sim \begin{pmatrix} \epsilon^4 & \epsilon^2 & 1 \\ \epsilon^2 & \epsilon^2 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{\epsilon^2}{\kappa} \begin{pmatrix} \kappa^2 & \kappa & 1 \\ \kappa & \kappa & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad (47)$$

If ϵ were zero, M would be rank-2. Since $\det(M) \sim \epsilon^2$, the third eigenvalue is of order ϵ^2 .

An alternative source of Majorana masses is instanton effects. M in this case is expected to be anarchic, with all eigenvalues of order unity [13].

Either way, the Majorana contribution might be relevant to the overall size of the neutrino masses, but does not help with the hierarchy. Only if Majorana masses aligned with Dirac masses to cancel the hierarchies in the Dirac masses could they help make the neutrino masses fit observations better. Instead, for the Majorana masses described above, the neutrino mass matrix $m^T M^{-1} m$ will inherit too large a large hierarchy from the Dirac mass m , in contradiction with experiment.

6.2.2 Multiple Intersection Points

If S', S'' were straight and completely orthogonal to S , there would be a single intersection point in $S \cap S' \cap S''$. However there could be more, and each would contribute to the Yukawa coupling NHL . From the point of view of the matter curves, having multiple triple-intersection points looks extremely nongeneric. However, since the curves arise from pairwise intersections of surfaces, they can't be moved around independently and this situation is perfectly natural and stable under perturbations.

This contrasts with the quark sector, where a triple intersection of curves $\Sigma_{H_d} \cap \Sigma_Q \cap \Sigma_D$ inside S is already nongeneric, relying on the assumption of an enhancement to a higher rank gauge symmetry at the intersection point. There's no reason to expect that the curves $\Sigma_{H_d}, \Sigma_Q, \Sigma_D$ might join again at a different symmetry enhancement point. So the prediction of a rank-one Yukawa should be robust for the quarks. However, the fact that neutrinos are gauge singlets implies that the geometry giving rise to their interactions is qualitatively different.

Interpreting the Yukawas in terms of the enhanced gauge symmetry, we are saying the quark Yukawas require an enhancement to an exceptional group, whereas the lepton Yukawa

¹²Another recently considered possibility is that right-handed Majorana masses are KK masses [15].

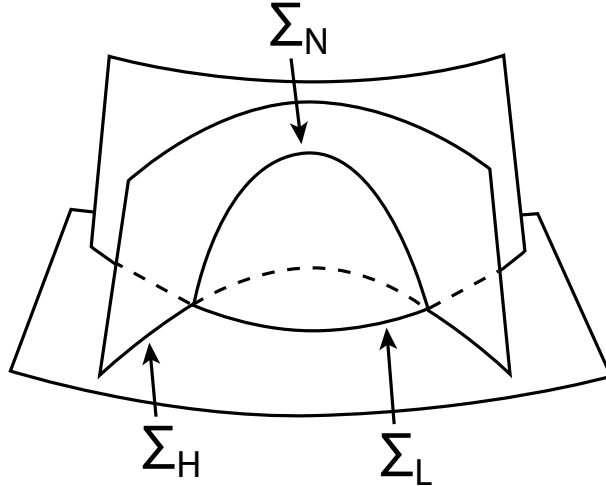


Figure 4: An approximately rank-2 Yukawa involving a gauge singlet.

does not. Although locally one might generate an exceptional group, it is clearly less generic for the global geometry to allow two enhanced symmetry points. There is no such nongeneric situation required for the neutrino mass.

The Dirac mass resulting from multiple intersections of S, S', S'' is then a sum of (approximately) rank-1 matrices m_q for each point $q \in S \cap S' \cap S''$. With two intersection points, perhaps the most likely of the possibilities, we generically get a rank-2 matrix. This can accommodate either a normal or inverted hierarchy. For instance, if q_1 is near q_2 along the curves $\Sigma_L, \Sigma_H, \Sigma_N$, then m_{q_1} and m_{q_2} will be approximately aligned, since the appropriate choice of orthonormal bases f^i on each curve is similar at q_1 and at q_2 (as in Section 5). The degree of alignment depends on the distance $|q_1 - q_2|/R = \delta$ along the matter curves. Generically, we expect $m_{q_1} + m_{q_2}$ to have eigenvalues of order $1, \delta^2, \epsilon^2$. That is, the second eigenvalue is set by the separation δ , and the third eigenvalue is set by the flux ϵ^2 , and vanishes when the flux goes to zero. Taking $\delta \approx 1/\sqrt{5}$ produces a viable normal hierarchy. Notice that ϵ^2 is smaller than the quark mass hierarchy because although the natural basis for neutrino wavefunctions might be of the form $1, z, z^2$ near the first Yukawa, there is no reason for this to be true (given our choice of z) at the second Yukawa. Whereas in the quark case, both left- and right-handed wavefunctions simultaneously lead to suppression, in this case, there is only one source of suppression.

So if q_1 and q_2 are not close, so that m_{q_1} and m_{q_2} are generically misaligned, then we expect two order one eigenvalues, and a third of order ϵ^2 . When the large eigenvalues are very nearly degenerate, this could reproduce the mass spectrum required for an inverted hierarchy. In a slightly nongeneric case, the two large eigenvalues might differ by a factor of five and produce a normal hierarchy with the lowest eigenvalue much smaller than the other two.

In either of these cases, if N gets an anarchic Majorana mass from instanton effects, the resulting neutrino masses will still be approximately rank-2, with one small eigenvalue of

order $\epsilon^2 \sim \alpha_{\text{GUT}} \sim \frac{1}{25}$ relative to the biggest.¹³

So it seems a fairly robust prediction of this scenario is a rank-2 mass matrix for the neutrinos at leading order. Conservatively, the third eigenvalue, either in the normal or inverted hierarchy, should be at least a factor of ten smaller than the largest eigenvalue. In either normal or inverted scenarios, this sets the overall scale for neutrino masses – not just their mass difference.

It is of interest to ask whether the approximately rank-2 form for the neutrino mass matrix could be tested. The best measurement in this regard could be neutrinoless double beta decay (though possible cosmological measurements might ultimately test the overall neutrino mass scale $m_\nu = m_1 + m_2 + m_3$ down to 0.04 eV [16, 17]). The matrix element for the decay is proportional to the element $m_{\beta\beta}$ in the neutrino mass matrix. We have [18]

$$|m_{\beta\beta}| \approx |\cos^2(\theta_{12})m_1 + e^{i\alpha_{12}}\sin^2(\theta_{12})m_2 + e^{i\alpha_{13}}\sin^2(\theta_{13})m_3| \quad (48)$$

where α_{12} and α_{13} are possibly nonvanishing Majorana phases.

The size of $|m_{\beta\beta}|$ depends on whether we have a normal or inverted hierarchy. Assuming near-vanishing m_1 , with the normal hierarchy, the contribution comes primarily from m_2 . But for nonzero θ_{13} , there can be a reasonably large correction since the largest eigenvalue $.046 \text{ eV} < m < .056 \text{ eV}$ is significantly bigger than the middle eigenvalue $.0088 \text{ eV} < m < .0091 \text{ eV}$.

We then find

$$0 \text{ eV} < |m_{\beta\beta}| < .005 \text{ eV} \quad (49)$$

where most of the uncertainty comes from the unknown phase $\alpha_{12} - \alpha_{13}$. If m_1 is nonvanishing, there will be an additive contribution. A reasonable estimate for this contribution in our scenario is of order $\alpha_{\text{GUT}}m_3 \approx 0.002 \text{ eV}$. If, on the other hand, $m_1 \approx m_2$, as might be more generically the case (that is, not in our models), we would expect an additional contribution of order 0.01 eV (since there is a linear contribution from m_1 not suppressed by $\sin^2\theta_{12}$). Distinguishing the small m_1 value of our model from the generically larger one requires a level of precision of order 0.01 eV, which is clearly beyond the capacity of any planned experiment. This nonetheless provides a useful target if we are to ultimately distinguish an exceptionally small lightest eigenvalue.

Cosmological bounds might then be the best way to detect a small third eigenvalue in the case of a normal hierarchy, since we'd expect $m_\nu \sim \sqrt{\Delta m_{32}^2} \sim 0.05 \text{ eV}$ (Fig. 6), which could be within reach of future studies [17].

In the case of the inverted hierarchy, the two larger masses are both in the range $.046 \text{ eV} < m < .056 \text{ eV}$. We then find $m_{\beta\beta}$ ranges from

$$0.013 \text{ eV} < |m_{\beta\beta}| < .056 \text{ eV} \quad (50)$$

¹³This same prediction for the smallest eigenvalue was also recently derived from a very different geometrical setup for neutrinos in F-theory, though only with a normal hierarchy [15]. Our upper bounds on measurements of $m_{\beta\beta}$ and m_ν will apply to their scenario as well.

with most of the uncertainty from α_{12} . The correction from m_3 would be at most about $\sin^2(\theta_{13})m_1/10 < 0.003 \text{ eV}$. In this case the leading correction from a larger value for the smallest eigenvalue than the $\alpha_{\text{GUT}}m_1$ expected in our model would come from larger values for both m_1 and m_2 , since only mass differences are known. So a substantially larger value for $m_{\beta\beta}$ than the minimal value could rule out this model. However, with a measurement in the above range it will be difficult to determine if the smallest eigenvalue is bigger than we would expect in our model. The value, though consistent with an inverted hierarchy, would not necessarily be precise enough to determine the overall mass scale with sufficient accuracy to pin down the mass of the lightest eigenvalue, especially until the phase α_{12} is known.

Once again, cosmological bounds could test our model, since an inverted hierarchy with a small third eigenvalue predicts $m_\nu \sim 2\sqrt{\Delta m_{32}^2} \sim 0.1 \text{ eV}$, in reach of future studies (Fig. 6).

It's also clear that these models can be ruled out by planned neutrinoless double β decay experiments. The largest possible value for $m_{\beta\beta}$ that we predict is .056 eV. Any larger value would indicate the overall scale of the neutrino masses is bigger than would be expected from this rank-2 matrix form (Figure 5).

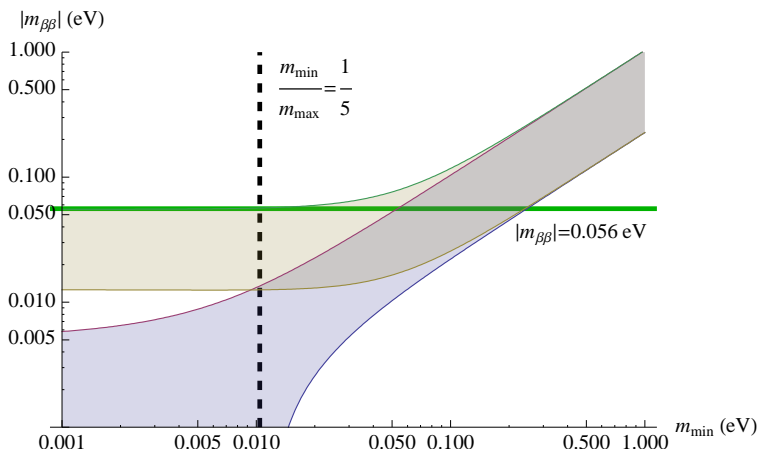


Figure 5: Possible values of $|m_{\beta\beta}|$ versus the smallest mass eigenvalue m_{\min} . The brown shaded region corresponds to the inverted hierarchy, while the blue corresponds to the normal hierarchy. The uncertainty is from a combination of Majorana phases and uncertainty in the known values of $\Delta m_{12}^2, \Delta m_{23}^2$, and the mixing angles. We predict $\frac{m_{\min}}{m_{\max}} \approx \alpha_{\text{GUT}} \approx \frac{1}{25}$, so a conservative upper bound for m_{\min} would be $\frac{m_{\min}}{m_{\max}} = \frac{1}{5}$, about 5 times larger allowing for unknown order unity factors. This yields a rough upper bound of $|m_{\beta\beta}| < 0.056 \text{ eV}$ (green line, above). A higher measured value for $|m_{\beta\beta}|$ would rule out our model.

7 Conclusions

Given the many potential routes for getting from string theory to the Standard Model, it is worthwhile to investigate qualitatively new models that might give novel insights into perplexing puzzles such as the hierarchy or flavor problems. In some cases the models merely

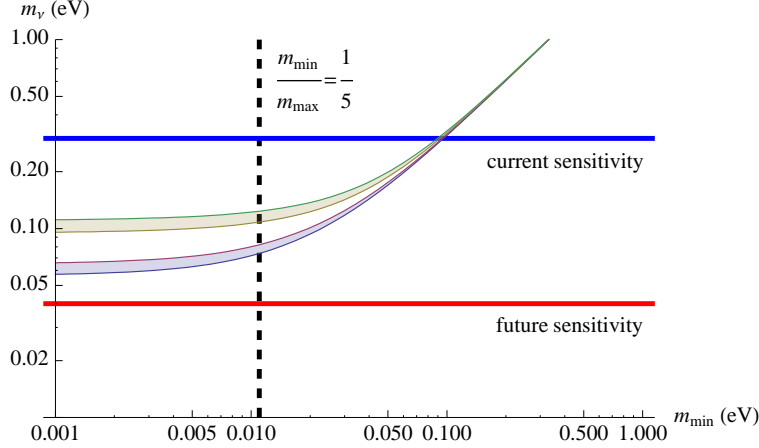


Figure 6: Values of the sum of neutrino masses m_ν versus the smallest eigenvalue, m_{\min} , for either the normal hierarchy (blue) or inverted hierarchy (brown). Conservatively, our model restricts us to the region $\frac{m_{\min}}{m_{\max}} \lesssim \frac{1}{5}$ (left of the dotted line), and thus gives a reasonably clear prediction for m_ν . For either the normal or inverted hierarchy, these predictions can't be tested with current precision but should be within reach of future studies [17].

implement known mechanisms, but sometimes they introduce genuinely new ideas. In other cases, such as this one, the theory falls somewhat in between. Technically the models we describe don't necessarily have mechanisms that cannot be accounted for with symmetries and additional heavy fields. Nonetheless the models from an effective theory viewpoint might be cumbersome or somewhat artificial.

In this work we have shown the underlying mechanism that allows F-theory models to reproduce the flavor structure of the Standard Model. We have also shown how to obtain the correct pattern of masses and mixings in the neutrino sector. We have furthermore shown that an interesting prediction seems to be a dominantly rank-2 matrix for the neutrinos and we have shown how this prediction can give testable consequences.

8 Acknowledgements

We'd like to thank Matthew Buckley, Clay Cordova, Jonathan Heckman, Joe Marsano, Yasinori Nomura, Sakura Schafer-Nameki, John Preskill, Jihye Seo, Sean Tulin, Mark Wise, and Cumrun Vafa for useful comments on this work. LR thanks the California Institute of Technology and the Moore Fellowship Program for their hospitality while this work was completed. This research was supported in part by NSF grant PHY-055611.

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